

High-Capacity Tube Network Design using the Hough Transform

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A new airspace design concept, tube networks, could enable high-density operations with less air traffic control workload. To construct tubes optimally, it is necessary to identify the commonality of flight trajectories. This paper proposes a new strategy to cluster great circle flight trajectories for forming tubes. The Hough Transform is applied to identify groups or clusters of great circle trajectories that could form the tube networks. The Genetic Algorithm is then applied to optimize the tube network. Results show that small deviations from great circle routes could yield tubes that accommodate significant traffic levels within feasible computational time.

I. Introduction

The predicted growth in traffic demand over the next 20 years could result in increased congestion and delays in the National Airspace System (NAS). Under the Dynamic Airspace Configuration (DAC) research sponsored by NASA,¹ two major concepts, dynamic resectorization and corridors-in-the-sky, or tube networks, have been proposed to ease congestion and delay. Developing tube networks is a promising concept due to its function of enabling high-density operations with less Air Traffic Control (ATC) workload.² Other potential benefits may include lower Traffic Flow Management (TFM) restrictions, better predictability of flight operations, and fuel savings due to shorter routes.

Impenetrable tubes may result in additional costs for flights that need to be rerouted to avoid conflict with the traffic within the tubes. Thus, the number of tubes should not be too large. On the other hand, to be beneficial and efficient, the limited number of tubes should be able to accommodate large traffic levels and should not require tube users to fly too much extra distance to utilize the tube. To meet these three requirements: low numbers of tubes, large number of flights in the tubes, and low extra flight distances, it is important to observe and to identify the commonalities of the nationwide flight trajectories. If the number of flights that take common routes is large, constructing tubes based on these common routes will be valuable. Otherwise, tubes might not provide much advantage due to the confliction with the remaining traffic.

Several research efforts have focused on tube networks. Alipio et al.² and Yousefi et al.³ suggested tubes based on city pairs by investigating the potential placement of tubes between two major cities such as Chicago and New York. If tubes are constructed based on city pairs, only a small portion of traffic will benefit with a limited number of tubes. Table 1 shows the flights among the top 25 US airports versus the flights over the whole NAS on April 20, 2007. This network requires more than 200 links/tubes and can only serve at most 20% of the flights. Sridhar et al.⁴ proposed a tube network interconnecting airports in clusters seeded by major airports to impact a significant amount of traffic. Although approximately 90% of flights are included, fully utilizing the tubes may require too much extra flight distance for tube users, which could make this method prohibitive.

A new methodology that proposes a limited number of tubes base on observed patterns of flight trajectories is desired. In this work, consider trajectories that follow the great circle flight routes between city

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Table 1. Flights Among Top 25 US Airports

Altitude (ft)	Flights among Top 25 airports	Flights over US	Percentage
>0	7,117	60,362	11.8%
>12,000	6,870	47,313	14.5%
>18,000	6,677	41,229	16.2%
>24,000	6,313	35,590	17.7%
>29,000	5,914	29,887	19.8%
>35,000	3,688	17,916	20.6%

pairs. Several image/signal processing techniques for grouping flight patterns were investigated. Robelin et al.⁵ developed an aggregation technique based on Generalized Principal Component Analysis (GPCA)⁶ for constructing a model of air traffic flow directly from Aircraft Situation Display to Industry (ASDI) data. Martinez et al.⁷ also used this aggregation technique to build a network flow graph. In these works, the authors used GPCA to get dominant directions at designated fixes from historical trajectories composed of series of waypoints/fixes. In this work, only origin and destination are considered for the great circle flight trajectory. Therefore, dimension reduction techniques like GPCA may not help. In computer vision and pattern recognition, the Hough Transform⁸⁻¹⁰ has been widely used for detecting lines, curves, or even geometric shapes that can be defined by parametric equations. Based on the point-line duality, the Hough Transform can be used to transform great circle trajectories to points in another space. Points that are close to each other will likely correspond to great circle trajectories that are close enough to form a tube. Thus, the Hough Transform is a good candidate for clustering great circle flight trajectories.

This paper presents a new methodology based on the Hough Transform and Genetic Algorithm to observe and cluster common great circle flight routes. This work provides the basis for future construction by proposing the potential tubes. The results show that a limited number of tubes in the national airspace can accommodate significant flights with low extra flight distances for tube users. Furthermore, dynamic tube recognition can be fulfilled by executing such a procedure periodically.

II. Modelling and Architecture of Designing Tubes

In this section, the data used were discussed. The procedures for identifying and proposing the potential tubes that eventually compose the networks are briefly introduced as well.

A. Data Acquisition

Traffic data for this work were obtained from the FAA's Aircraft Situation Display to Industry (ASDI) for the entire day of April 20, 2007. The ASDI file for this day contains 62,143 flights. Because only the continental US is of interest, a rectangular bounding box surrounding the continental US is formed: the latitude range is $[17.2^\circ N, 50.0^\circ N]$ and longitude range is $[225.8^\circ E, 296.7^\circ E]$. For each flight, only the origin and destination are used for defining great circle flight routes. Without loss of generality, only flights with tracks above FL290 are taken into account. During preprocessing, the great circle flight trajectories are determined, and those trajectories that do not overfly the continental US region are removed. These preprocessing steps result in a final total of 29,629 flights to be considered.

B. Procedure

The procedure to identify and generate tubes is shown in Figure 1. First, the preprocessed great circle trajectories are transformed into points in Hough space. Each point corresponds to one great circle flight route. Next, initial clustering is performed by the voting scheme on the basis of those points, which is essentially a grid based clustering. Because the criterion for clustering the trajectories is minimal excess flight distance instead of the Euclidian distance, the flights are removed from their clusters if there is too much extra flight distance. Based on these initial clusters, the Genetic Algorithm is applied to refine the

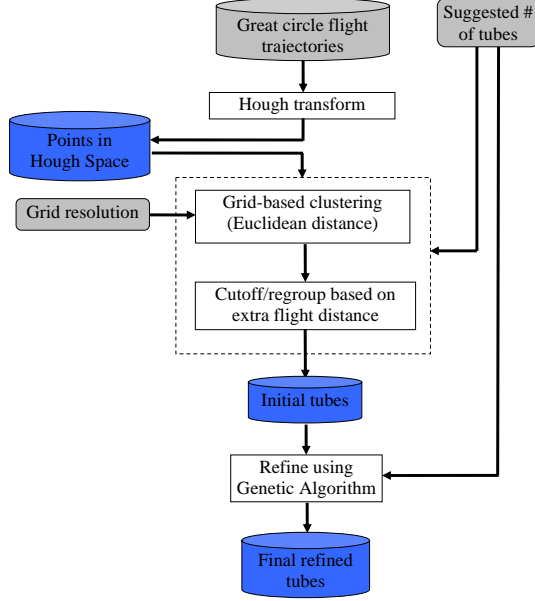


Figure 1. Procedure to identify and generate tubes

results by moving the centers of tubes to seek better clustering.

III. Methodology and Results

In this section the Hough Transform and its parameterization are described first. After that, the initial clustering based on the voting scheme for the points in the Hough space is presented. On the basis of the initial clusters, the Genetic Algorithm is applied to improve the design.

A. Hough Transform for Great Circle Routes

Polar representation of a line is referred to Hough Transform in image processing literature.⁹ The equation to a line with the shortest distance ρ to the origin and an angle θ between its normal and the x axis is given by:

$$\rho = x \cos \theta + y \sin \theta \quad (1)$$

The (x, y) are in Cartesian coordinates, and the (ρ, θ) are in the Hough space. This property is also called point-line duality.

Although a great circle line lies in 3-dimensional space, it actually has only 2 degrees of freedom. Thus, it is suitable to apply the Hough transform. Given a flight with the origin at (δ_o, λ_o) and the destination at (δ_d, λ_d) , assuming the Earth is an ideal sphere, the Cartesian coordinates will be:

$$\vec{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = R \cdot \begin{bmatrix} \cos \lambda_i \cdot \cos \delta_i \\ \sin \lambda_i \cdot \cos \delta_i \\ \sin \delta_i \end{bmatrix} \quad (2)$$

where $i = o, d$ representing the origin and destination, respectively. R is the radius of the Earth, δ and λ are latitude and longitude, respectively, and x, y, z are the Cartesian coordinates. Any point $\vec{r} = [x, y, z]^T$ on this great circle line should satisfy:

$$\begin{cases} \vec{r}_o \times \vec{r}_d = \vec{r} \times \vec{r}_d \\ \|\vec{r}\| = R \end{cases} \quad (3)$$

Just as with straight-lines, to transform the great circle lines a reference point has to be defined along with a reference axis. The reference point can be put anywhere inside of the bounding box, and the reference axis can point to any direction. Though different references generate different Hough transform results, they do not affect the nature of clustering as long as the references are fixed during the whole process. In this work, the reference point is set at $[33.6^\circ N, 261.3^\circ E]$ - the center of the defined rectangular region, while the reference axis is defined to point the East. Figure 2 shows how ρ is defined as the shortest distance between the reference point and the great circle route and how θ is defined as the cross angle between the reference axis and the tangent vector at the reference point, which is aligned with ρ . Here $\rho \in [0, \infty]$ and $\theta \in [0, 2\pi]$. Detailed derivation of Hough and DeHough transforms for great circle routes are presented in the Appendix.

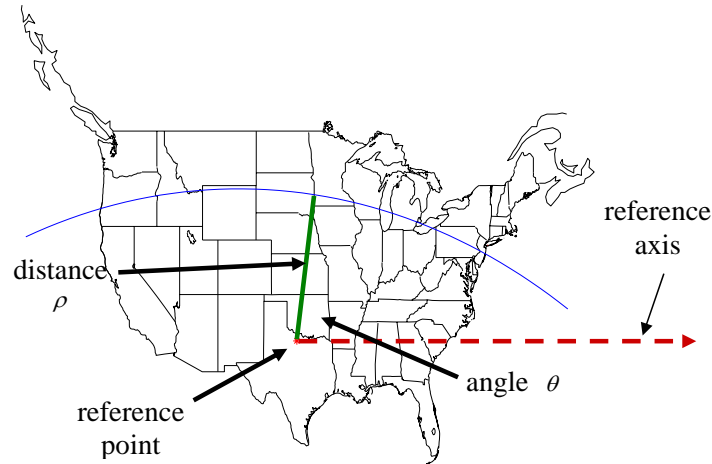


Figure 2. Hough Transform for Great Circle Routes

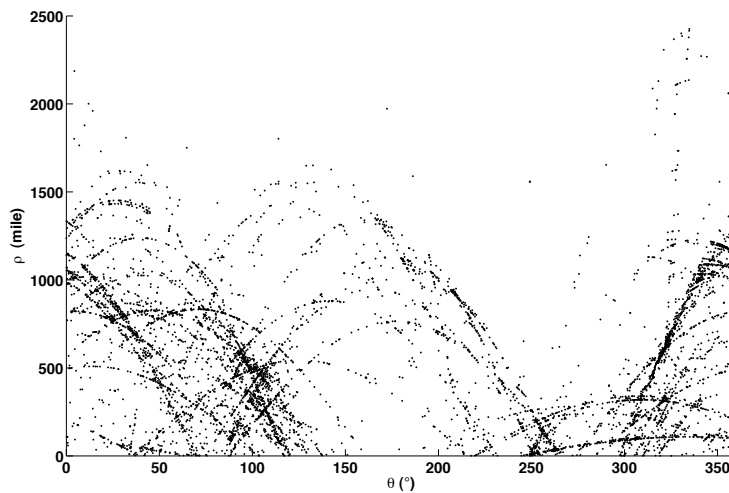


Figure 3. Transformed Trajectories as Points in Hough Space

Figure 3 shows the results after the Hough transform. The 29,629 great circle flight routes are transformed to points in Hough space. Some of them overlap each other. Apparently, the relationship among the trajectories can be easily told by the points in the Hough space. Notice how the points form many parabola-like curves in the Hough space. This phenomenon can be explained using the point-line duality. Each curve essentially implies an airport, and the points forming the curve correspond to all the trajectories going

through that airport. This supposition can be borne out by the following equation:

$$\rho = R \cdot \arctan\left[\tan\left(\frac{d}{R}\right) \cdot \cos(\gamma - \theta)\right] \quad (4)$$

where d is the shortest distance/great circle distance between an airport and the reference point. The variable γ is the angle between the line, which links the airport and the reference point, and the reference axis. The derivation can be found in the Appendix. Given an airport, the values of d and γ , are fixed. Because the radius of Earth R is a constant, parameters ρ and θ have a fixed relationship. For instance, ORD has $d = 826$ miles and $\gamma = 47.7^\circ$, while JFK has $d = 1451.8$ miles and $\gamma = 26.8^\circ$. As shown in Figure 3, all the trajectories going through these two airports fall on the yellow and green curves, respectively.

B. Initial Results

In image processing, the continuous (ρ, θ) Hough space is then quantized into suitable size $(\Delta\rho \times \Delta\theta)$ grids and each grid is associated with an element of an array called the accumulator array. After this, the lines are extracted by the voting scheme. In this work, similar grids are constructed in the Hough space to catch the potential high-density tubes. Without loss of generality, the number of tubes is set to 60 in this work. The resolutions are defined as $\Delta\rho = 50$ miles and $\Delta\theta = 10^\circ$. The 60 grids with the highest point density are shown as red rectangles in Figure 4. They accommodate 59% of the 29,629 flights. After the DeHough transform, these 60 grids become 60 tubes. A sample tube from one of these grids is presented in Figure 5(a). The red curve/tube is transformed from a midpoint which is the average over all points inside of the grid in the Hough space. The blue curves are the actual great circle flight routes that can utilize this tube.

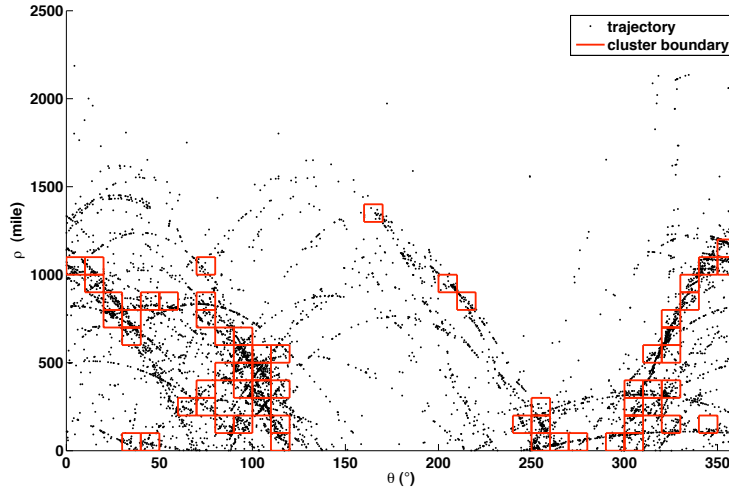


Figure 4. ORD and JFK in Hough Space and the Initial Top 60 grids based on Hough Transform

If the Hough coordinates of a flight trajectory are not exactly the same as the weighted center of its corresponding grid, there will be extra flight distance for the flight to use the designated tube. On the other hand, due to the finite length, the great circle routes have largely varied extra distances even if the coordinated in $(\rho - \theta)$ space are the same. Therefore, the extra distance, which is assumed to be the main concern of the tube users, should serve as the criteria for clustering instead of the Euclidean distance. The extra distance must be calculated in the original space, because the origin and destination are lost in the transformation. The extra flight distance, shown in Figure 5(b), is defined as a rate:

$$d_{extra} = \frac{d_1 + d_2}{D} \cdot 100\% \quad (5)$$

where d_i are the shortest distances to join or leave the tube, and D is the original great circle flight distance. Figure 5(b) demonstrates that if the flight with the blue trajectory utilizes the red tube, it will have d_{extra} percent of deviation from its shortest distance.

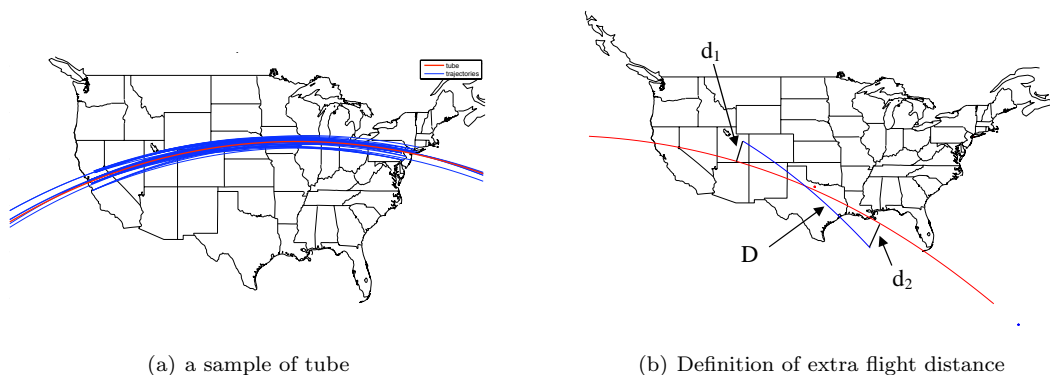


Figure 5. Tube and Extra distance

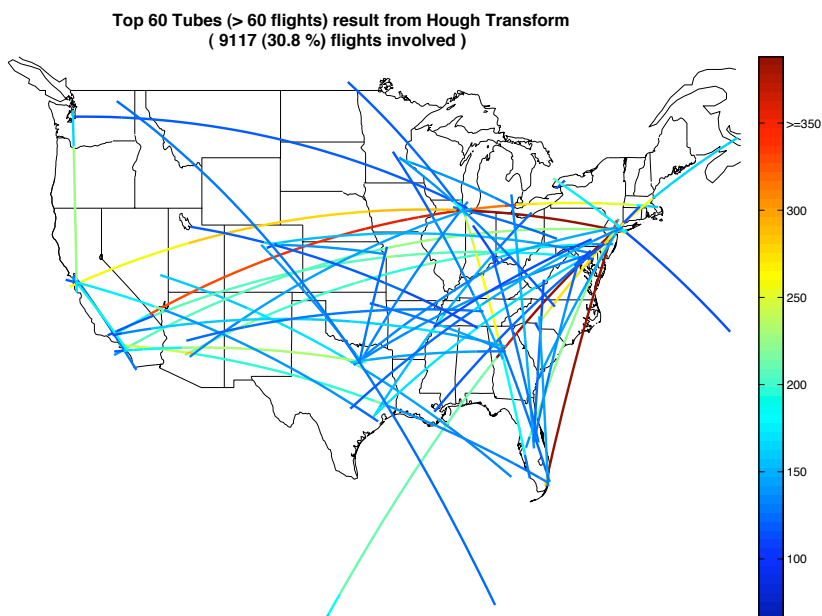


Figure 6. Top 60 tubes based on Hough Transform

Most of the clustering algorithms suitable for Euclidean distance will not be efficient for the extra flight distance due to the complexity of the relative positions with the extra flight distance. Even a stochastic-process-based optimization algorithm, like the Genetic Algorithm, cannot determine the optimal solutions in a feasible computational time without a good initial guess. The search will be exhaustive and the required computational time will make it prohibitive. Fortunately, clustering based on Euclidean distance in the Hough space should be rough estimate of the clustering based on the extra distance in the original space. For instance, if two trajectories are far away from each other in the Hough space, they most likely have large extra distance. On the other hand, if they are far from each other in the $\rho - \theta$ space, the probability of having low extra distance between them is low.

Thus, the clustering results based on Euclidean distance in the Hough space is utilized and the flights that have higher extra distance in original space than a defined threshold are removed. If the tolerable extra flight distance is defined as 5% of the shortest flight distance, the percentage of flights that top 60 tubes can hold drops from 59% to 31% . The large decrease indicates the difference between two clustering criteria. Figure 6 displays the top 60 tubes. In this figure, only the portions that have more than 60 flights are shown. A warmer color denotes higher density of the traffic, whereas a colder color represents lower density. The density of the traffic can be judged by the color bar.

C. Refinement using Genetic Algorithm

To maximize the number of benefited flights, an optimization algorithm is applied to these initial tubes. In this work, the Genetic Algorithm (GA) is used to optimize the clustering, because it has been widely used as a powerful optimization method.

The GA is a stochastic process which models two natural phenomena: genetic inheritance and Darwinian evolution.¹¹ It first creates a population of potential solutions. Each solution is called a “chromosome”, represented by a binary string of length $m = \sum_{i=1}^k m_i$, where k is the number of design parameters. The first m_i bits of the string correspond to the first design parameter, or “gene”. The next group with m_i bits will map to the second design parameter, and so on. In each generation, the population of chromosomes will be evaluated by using a cost function. The new population is selected with respect to the probability distribution based on fitness values. Finally, the chromosomes are altered in the new population by mutation and crossover operators.

In this work, the weighted centers of the grids in the Hough space are optimized. Because each center has two coordinates, ρ and θ , the number of optimization parameters will be 120. The clustering results from the Hough transform discussed above will serve as initial values for the GA. For the selection process inside of the GA, a roulette wheel with slots sized according to fitness is used. The crossover probability is 0.8 and mutation probability is 0.2. The population size is set to 200 and it will stop after 200 generations. The threshold for the extra flight distance is 5%.

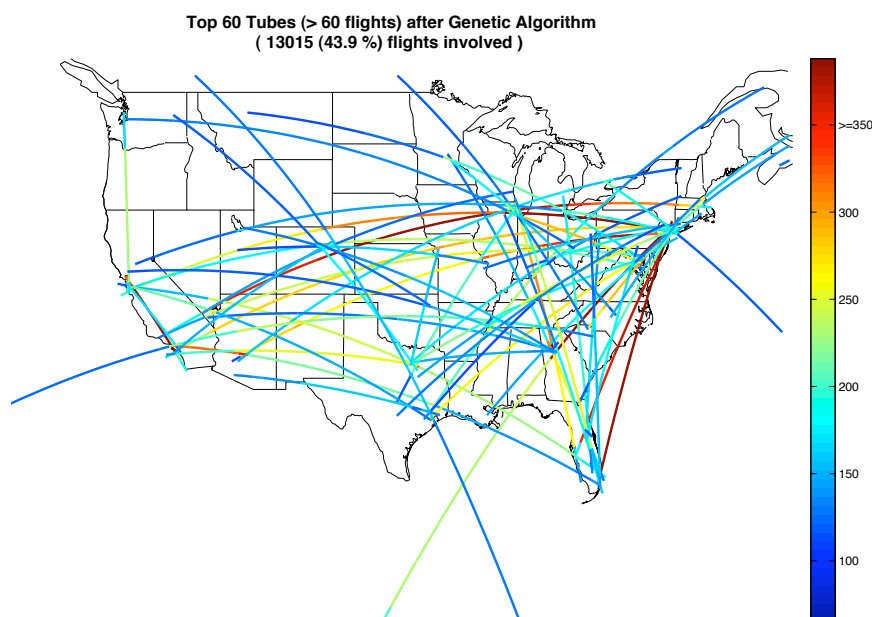


Figure 7. Top 60 tubes after refining using GA

After 200 generations, the percentage of flights that the top 60 tubes can accommodate increases from 31% to 44%. The grid centers did not change dramatically, which means the original clustering provided a good start. Figure 7 shows the final top 60 tubes. Only the portions which have more than 60 flights are displayed. On the platform of MacOS with Intel Core 2 Duo Processor 3.0 GHz, 200 generations takes around 10 hours with 4 threads working in parallel. This computation time can be shortened by lowering the number of generations or running the GA in more powerful parallel computing clusters.

D. Analysis

To get insight of operations in the tubes, the flights within the designed tubes are investigated. Figure 8 shows the flights between any two of the top 25 airports versus the rest in each tube. Similar to the observation in Sridhar’s work,⁴ it is noticed that flights among these airports do not account for a major portion of the total operations within the tubes. These results further testify the importance of building tubes upon the nature of flight trajectories versus major-city pairs.

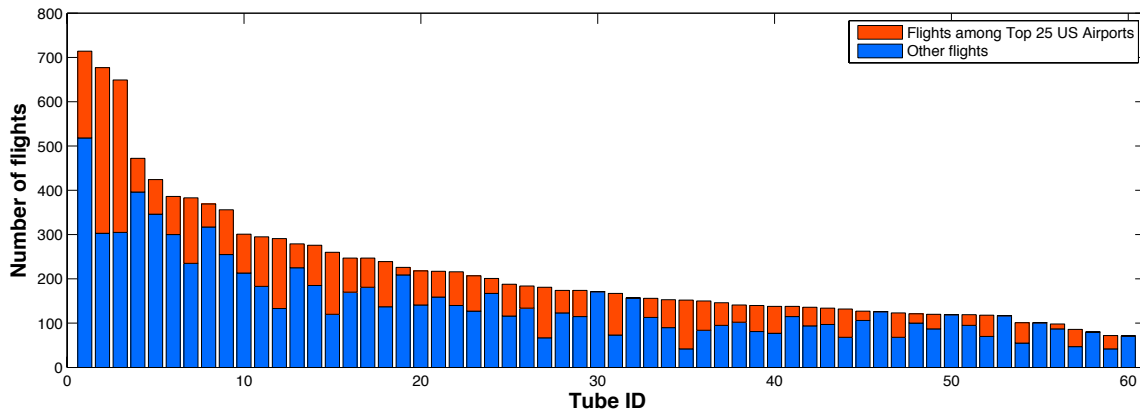


Figure 8. Percentage of flights among Top 25 US airports

This study focused on flights over FL290. Several other partitions are explored in a altitude sensitivity analysis. Based on a 5% deviation, Table 2 presents the results for all flights that are flying above a given altitude. The results show that flights at high altitudes have stronger tube conformance than the ones at low altitudes, which implies that it may be more valuable to construct tubes/corridors at high altitudes than at low altitudes.

Table 2. Flights Included in Tubes Given 5% Deviation

Altitude (ft)	Flights involved	Percentage
>0	17,274	29%
>24,000	14,485	41%
>29,000	13,015	44%
>35,000	8,658	49%

Table 3. Flights Included in Tubes Given varied Deviation

Deviation	Flights involved	Percentage
1%	3,692	13%
2%	6,736	23%
3%	8,998	30%
4%	11,265	38%
5%	13,015	44%
7%	16,085	54%
9%	18,666	63%
11%	20,407	69%

The sensitivity analysis can be conducted if the deviation constraints are varied. Table 3 presents the relationship between the deviation constraints and the number of flights included in the tubes based on flights over FL290. The results demonstrate that the more deviation allowed from the shortest distance, the more flights can be included into the tubes. This provides us a profile of the benefits pool of constructing tube network. Although the largest deviation can yield about 70% flights operated in the tubes, it should be carefully decided in practice and weighed against the cost of deviating.

IV. Conclusions

Identifying tubes that catch the commonalities of the shortest distance trajectories is necessary for constructing tubes/corridors-in-the-sky structure in future airspace. If the clustering is based on the city-pairs to cluster, only a small amount of traffic per tube might benefit. Building tube network clusters by seeding it with major-cities increases the benefit pool but requires too much extra flight distance for the tube users. Clustering the trajectories directly can identify the potential tubes that provide a compromise between the benefit pool and the extra flight distance required.

In this work, a new methodology is developed based on the Hough transform, which enables the clustering of great circle flight routes. Extra flight distance beyond the shortest flight distance serves as the criteria for clustering. There is no single existing clustering algorithm that can solve this problem efficiently, thus, the Hough Transform and Genetic Algorithm are combined. The Hough Transform generates good initial guesses, and thereafter, GA refines the tubes if the restriction of computational time allows. Results show that significant traffic would benefit from the tubes by only using a small number of tubes. Utilizing these tubes only requires five percent extra flight distances. The entire process could be achieved within a feasible computational time. This procedure could be directly utilized for dynamic tube design if the data are fed in during incremental time periods. The list of tubes proposed by our method can be used as a basis. With further constructional and operational analysis, the final tubes or network can be eventually constructed. This method can be applied to the wind optimal trajectory if the wind velocity is constant for the entire trajectory. In future work, efforts will be put on general wind optimal trajectories.

Appendix

As described in Figure 9, given a great circle flight route composed of two points/airports (\vec{u} , \vec{v}), to get the unique hough transform, a reference point \vec{p} is first configured, which has geographic coordinates, latitude (δ_p) and longitude (λ_p), and Cartesian coordinates $[p_1, p_2, p_3]^T$. In this work, the ρ , one parameter in Hough transform, is defined as the shortest distance between point \vec{p} and the given great circle on the sphere/earth surface. Assume the great circle plane Λ containing \vec{u} and \vec{v} has norm $\vec{n}_\Lambda [n_{\Lambda 1}, n_{\Lambda 2}, n_{\Lambda 3}]^T$. If we create another great circle plane Θ , which includes reference point \vec{p} and is perpendicular to existing plane Λ , then ρ will be the great circle distance between \vec{p} and the closer intersection point \vec{f} ($[\delta_f, \lambda_f]$ and $[f_1, f_2, f_3]^T$). Furthermore, we define the tangent vector \vec{E}_p at point \vec{p} , which points East, to be the base direction. Thus, another parameter in Fough transform θ is defined as the cross angle between vector \vec{E}_p and the tangent vector \vec{F}_p at point \vec{p} , which lies in the great circle plane Θ .

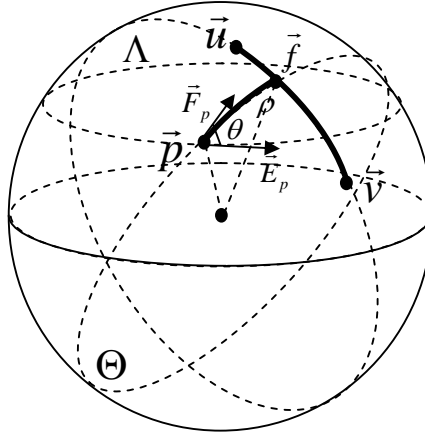


Figure 9. Calculation of Hough Transform

A. Calculation of ρ in Hough Transform

To compute ρ , we first need to find the intersection points mentioned above. Then, the closer one is chosen. Assume the points are $\vec{f}_A (\delta_{f_A}, \lambda_{f_A})$ and $\vec{f}_B (\delta_{f_B}, \lambda_{f_B})$. δ and λ should satisfy the following equations:

$$\cos \lambda \cdot n_{\Lambda 1} + \sin \lambda \cdot n_{\Lambda 2} + \tan \delta \cdot n_{\Lambda 3} = 0 \quad (6a)$$

$$\begin{bmatrix} p_2 \cdot \tan \delta - p_3 \cdot \sin \lambda \\ p_3 \cdot \cos \lambda - p_1 \cdot \tan \delta \\ p_1 \cdot \sin \lambda - p_2 \cdot \cos \lambda \end{bmatrix} \cdot \begin{bmatrix} n_{\Lambda 1} \\ n_{\Lambda 2} \\ n_{\Lambda 3} \end{bmatrix} = 0 \quad (6b)$$

If $p_2 \cdot n_{\Lambda 1} - p_1 \cdot n_{\Lambda 2} \neq 0$, from Eqn. (6b), it can be derived:

$$\tan \delta = \frac{-\sin \lambda (p_1 n_{\Lambda 3} - p_3 n_{\Lambda 1}) - \cos \lambda (p_3 n_{\Lambda 2} - p_2 n_{\Lambda 3})}{(p_2 n_{\Lambda 1} - p_1 n_{\Lambda 2})} \quad (7)$$

by substituting Eqn. (7) in Eqn. (6a) we have:

$$[(p_2 n_{\Lambda 1} - p_1 n_{\Lambda 2}) n_{\Lambda 1} - (p_3 n_{\Lambda 2} - p_2 n_{\Lambda 3}) n_{\Lambda 3}] \cos \lambda + [(p_2 n_{\Lambda 1} - p_1 n_{\Lambda 2}) n_{\Lambda 2} - (p_1 n_{\Lambda 3} - p_3 n_{\Lambda 1}) n_{\Lambda 3}] \sin \lambda = 0 \quad (8)$$

By defining:

$$A = (p_2 n_{\Lambda 1} - p_1 n_{\Lambda 2}) n_{\Lambda 2} - (p_1 n_{\Lambda 3} - p_3 n_{\Lambda 1}) n_{\Lambda 3} \quad (9a)$$

$$B = (p_2 n_{\Lambda 1} - p_1 n_{\Lambda 2}) n_{\Lambda 1} - (p_3 n_{\Lambda 2} - p_2 n_{\Lambda 3}) n_{\Lambda 3} \quad (9b)$$

Given the reference point \vec{p} and US region interested, we have:

$$A^2 + B^2 > 0 \quad (10)$$

Eqn 8 then becomes:

$$\sin(\lambda + \beta) = 0 \quad (11)$$

where:

$$\cos \beta = \frac{A}{\sqrt{A^2 + B^2}} \quad (12a)$$

$$\sin \beta = \frac{B}{\sqrt{A^2 + B^2}} \quad (12b)$$

Eventually, we get: $\lambda = k\pi - \beta, k = [\dots -1 \ 0 \ 1 \ \dots]$, where $\lambda \in [0, 2\pi]$. Since there only exists two values between 0 and 2π , define them to be λ_1 and λ_2 , respectively.

If $p_2 \cdot n_{\Lambda 1} - p_1 \cdot n_{\Lambda 2} = 0$, from Eqn. (6b), we get:

$$-\sin \lambda (p_1 n_{\Lambda 3} - p_3 n_{\Lambda 1}) - \cos \lambda (p_3 n_{\Lambda 2} - p_2 n_{\Lambda 3}) = 0 \quad (13)$$

By defining:

$$C = -(p_1 n_{\Lambda 3} - p_3 n_{\Lambda 1}) \quad (14a)$$

$$D = -(p_3 n_{\Lambda 2} - p_2 n_{\Lambda 3}) \quad (14b)$$

Again, given the reference point \vec{p} and the US region of interest, we have:

$$C^2 + D^2 > 0 \quad (15)$$

Then, rewrite Eqn 13 in simpler form:

$$\sin(\lambda + \gamma) = 0 \quad (16)$$

where:

$$\cos \gamma = \frac{C}{\sqrt{C^2 + D^2}} \quad (17a)$$

$$\sin \gamma = \frac{D}{\sqrt{C^2 + D^2}} \quad (17b)$$

Similarly, we have: $\lambda = k\pi - \gamma$, $k = [\dots -1 \ 0 \ 1 \ \dots]$, where $\lambda \in [0, 2\pi]$. Again, define the two values between 0 and 2π to be λ_3 and λ_4 , respectively.

Now, δ can be computed explicitly. If $n_{\Lambda 3} \neq 0$, from (6a), we can derive:

$$\delta_i = \arctan\left[-\frac{\cos \lambda_i n_{\Lambda 1} + \sin \lambda_i n_{\Lambda 2}}{n_{\Lambda 3}}\right] \quad (18)$$

where $i = 1, 2, 3, 4$.

If $n_{\Lambda 3} = 0$, then the great circle route should have constant longitude, which means the latitudes of intersection point should be equal to δ_p .

Finally, the latitude and longitude of the intersection point for shortest distance ρ can be concluded:

$$(\delta_f, \lambda_f) = \begin{cases} \underset{\substack{\delta \in [\delta_1, \delta_2] \\ \lambda \in [\lambda_1, \lambda_2]}}{\operatorname{argmin}} [great_circle_dist((\delta, \lambda), \vec{p})] & \text{if } p_2 \cdot n_{\Lambda 1} - p_1 \cdot n_{\Lambda 2} \neq 0 \text{ and } n_{\Lambda 3} \neq 0, \\ \underset{\lambda \in [\lambda_1, \lambda_2]}{\operatorname{argmin}} [great_circle_dist((\delta_p, \lambda), \vec{p})] & \text{if } p_2 \cdot n_{\Lambda 1} - p_1 \cdot n_{\Lambda 2} \neq 0 \text{ and } n_{\Lambda 3} = 0, \\ \underset{\substack{\delta \in [\delta_3, \delta_4] \\ \lambda \in [\lambda_3, \lambda_4]}}{\operatorname{argmin}} [great_circle_dist((\delta, \lambda), \vec{p})] & \text{if } p_2 \cdot n_{\Lambda 1} - p_1 \cdot n_{\Lambda 2} = 0 \text{ and } n_{\Lambda 3} \neq 0, \\ \underset{\lambda \in [\lambda_3, \lambda_4]}{\operatorname{argmin}} [great_circle_dist((\delta_p, \lambda), \vec{p})] & \text{if } p_2 \cdot n_{\Lambda 1} - p_1 \cdot n_{\Lambda 2} = 0 \text{ and } n_{\Lambda 3} = 0, \end{cases} \quad (19)$$

Where the operator *great_circle_dist* computes the great circle distance between the pair of geographic coordinates.

B. Calculation of θ in Hough Transform

Next, the cross angle θ between vector \vec{E}_p and \vec{F}_p is computed. Assuming the normal of the great circle plane Θ is \vec{n}_{Θ} , where:

$$\vec{n}_{\Theta} = \vec{n}_{\Lambda} \times \vec{p} \quad (20)$$

the vector \vec{F}_p can be computed by:

$$\vec{F}_p = \vec{n}_{\Theta} \times \vec{p} \quad (21)$$

Assuming the great circle plane that \vec{E}_p lies in is plane Γ and its normal is \vec{n}_{Γ} (actually normalized form is $[0, 0, 1]^T$), by defining the projection of \vec{p} in the horizontal plane, which contains the equator, as $\vec{p}'[p_1, p_2, 0]^T$, we get:

$$\vec{E}_p = \vec{n}_{\Gamma} \times \vec{p}' \quad (22)$$

Then, the θ' within $[0, \frac{\pi}{2}]$ is computed:

$$\theta' = \arccos\left(\frac{\vec{E}_p \cdot \vec{F}_p}{|\vec{E}_p| |\vec{F}_p|}\right) \quad (23)$$

By checking the relative position between point \vec{p} and \vec{f} , we can finally get θ explicitly.

C. Dehough Transform

Given a Hough transform point with θ and ρ , by simply reversing the procedures of the above Hough Transform with the same reference point, reference direction and interested region, we can compute the great circle flight route with two end points on the boundary of the defined region.

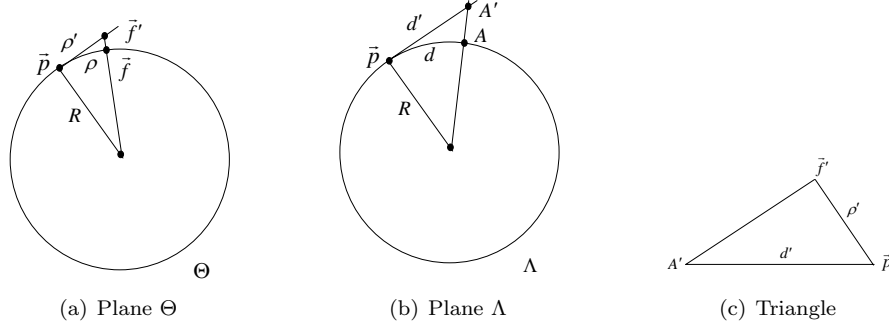


Figure 10. Calculation of Relationship for Trajectories via An airport

D. Relationship Among the Trajectories Going Through An Airport

Defining a reference point \vec{p} (δ_p, λ_p) and axis pointing to East, and specifying an airport \vec{A} (δ_A, λ_A), we could find the relationship between ρ and θ for any great circle routes going through the airport.

For any great circle route via airport \vec{A} , suppose f is the perpendicular foot on the great circle line, which makes $\widehat{pf} \perp \widehat{Af}$. Define the plane that is tangent to the earth at point \vec{p} as plane Π . Assuming the origin of earth is O , on plane Π , define that A' and f' are the points collinear with \vec{OA} and \vec{Of} , respectively. For the right triangle $\triangle A'f'P$, we have:

$$\rho' = \overline{pf'} = R \cdot \tan \gamma = R \cdot \tan\left(\frac{\rho}{R}\right) \quad (24)$$

and

$$d' = \overline{pA'} = R \cdot \tan\left(\frac{d}{R}\right) \quad (25)$$

Assuming the great circle distance/shortest distance between \vec{p} and \vec{A} is d , and the angle between the norm of route and the reference axis is γ , the relationship between ρ and θ can be expressed as:

$$\rho' = d' \cdot \cos(\gamma - \theta) \quad (26)$$

\Rightarrow Eqn. 4

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